CALCULATING INTERNAL FORCES FROM ATYPICAL STRAIN READINGS

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ABSTRACT

A valuable objective of statically load testing deep foundations is to determine load-transfer response. Integral to this objective is determining internal forces at various elevations within the deep-foundation element. These internal forces are often calculated from strain readings (strain instrumentation does not determine internal forces directly). Strain readings can sometimes yield atypical results, with multiple strain gauges at a given strain-gauge level ("SGL") (i.e., the same elevation) providing significantly different individual readings, or the deep-foundation element changing effective rigidity during a static load test. A case history will be presented where a bi-directional static load test was conducted on a test shaft with strain gauges affixed to a center core beam. At multiple SGLs, the case-study test measurements indicated both: a) atypical strain readings occurred simultaneously, and b) effective rigidity changed during the test. The strain readings were evaluated using several methods, including the American Concrete Institute formula and the Incremental Rigidity ("I.R.") method. In addition, the I.R. method was employed to interpret the change in effective rigidity at the individual SGLs during the test. The resulting load-transfer responses are reasonable, and so exhibit the usefulness of applying the I.R. method to atypical strain readings, thus demonstrating a means of interpreting internal forces at SGLs which might otherwise have to be disregarded (i.e., a means of "salvaging" an SGL).

Keywords: Incremental Rigidity method, instrumented static load test, internal force, load transfer, static load test, strain gauges, strain readings

INTRODUCTION

Static load tests have an important role in the design and construction of deep foundations of all types: driven piles, augered cast-in-place ("ACIP") piles, drilled displacement piles ("DDP"), helical piles, drilled shafts, etc. For simplicity, all deep foundation types will be referred to herein as piles. Additionally, the types of static load tests addressed herein are axial, both head-down and bi-directional compressive, and tensile. The usefulness of static load tests, particularly in the design phase, is enhanced by determining load-transfer response during the test. Load-transfer response refers to the manner in which internal pile forces are transferred into the surrounding geomaterial, with the definitive result being the magnitudes of mobilized unit shaft resistances along the pile length versus relative soil-pile movement. Integral to this objective is determining internal forces at various locations within the deep-foundation element.

Load-transfer determinations can be obtained using several different types of instrumentation; two common types are telltales and strain gauges (Brown et al. 2018). Neither instrumentation type measures internal forces directly. When detailed load-transfer data are desired, telltale measurements alone are insufficient (Hannigan et al. 2016). Weldable strain gauges can be used on steel pipe and H-piles, and "sister-bar" strainmeters or concrete-embedment strain gauges can be installed in concrete(d) or grouted piles.

Embedded strain gauges are generally treated as reliable (even if unbeknownst they are not) in obtaining measurements characteristic of foundation response to applied load; converting strain readings to internal

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pile forces is not necessarily straightforward, as it is a function of the pile's physical characteristics at the strain measurement location. This reality is at best misrepresented, and at worst misunderstood, by the notion that strain gauges measure internal forces (rather than provide readings that are an intermediate step in calculating internal forces). The physical characteristics required to convert strains to internal forces can vary by location within a pile, and during a static load test. Some physical characteristics can be measured, but are more often assumed, assigned presumptive values, estimated, based on constitutive relationships, or back-calculated. The methods used to determine a pile's physical characteristics can introduce significant error into internal pile force calculation, and therefore into determined mobilized unit shaft resistances.

Internal pile forces, F_i, at each strain-gauge level ("SGL") have conventionally been calculated using the strain reading (typically an average, if the SGL contains multiple strain gauges), ε_i, and the product of the pile's composite-section elastic modulus, E_i, and total cross-sectional area, A_i, by the following relationship (Greenspan 1943 & 1946; Pelecanos et al. 2017; Vable 2008):

$$F_i = E_i A_i \varepsilon_i \qquad [1]$$

The Incremental Rigidity ("I.R.") method provides a means of quantitatively back-calculating a pile's physical characteristics (i.e., EA) from load-test measurements, and therefore more direct conversion of strain measurements to calculated internal forces (Komurka & Moghaddam 2020; Komurka & Robertson 2020). This method may also be used quantitatively in conjunction with other strain interpretive methods, or qualitatively to identify atypical structural stress-strain response to applied load. Piece-wise linear functions may be developed where the rigidity is defined by multiple interpretative methods over the strain history during a load test (Robertson 2024; Sinnreich 2011).

INCREMENTAL RIGIDITY METHOD OVERVIEW

During a static load test, strain is the measured parameter in Eq. 1, and conversion of measured strains to calculated internal forces conventionally involves determining the composite-section elastic modulus and total cross-sectional area at each strain-gauge level. Komurka and Moghaddam (2020) presented the Incremental Rigidity method, which determines the product EA, the foundation's ***axial rigidity***, at each interpretable SGL. For simplicity, EA will be referred to herein as rigidity. The Incremental Rigidity method is based on the Tangent Modulus method (Fellenius 1989, 2001, and 2019; Salem & Fellenius, 2017), but instead of relating changes in stress to changes in strain to determine a modulus relationship, the I.R. method relates changes in test load to changes in strain to determine an internal force relationship. In this way, the I.R. method offers more direct conversion of strains to calculated internal forces.

Integral to the I.R. method is routine application to field load-testing results, strain-gauge instrumentation layout optimization, and improved load-testing protocol beneficial to yield interpretable results. After shaft resistance between the test-load source and a strain-gauge level is essentially fully mobilized, the quotient of change in test load divided by change in strain ($\Delta Q/\Delta\varepsilon$) plotted against total strain for that SGL resolves into a virtually straight line, sloping from a larger rigidity to a smaller one with increasing strain (Komurka & Moghaddam 2020). A $\Delta Q/\Delta\varepsilon$ plot example is presented in Fig. 1. Strain-gauge levels where shaft resistance between the test-load source and that SGL is not essentially fully mobilized are non-interpretable using the I.R. method; calculating internal forces at these SGLs is addressed in Komurka & Robertson (2020).

Qualitatively, deviations in $\Delta Q/\Delta\varepsilon$ over the test duration may indicate variations in pile rigidity at specific strain magnitudes at a specific strain-gauge level. For example, if full composite-section action is not exhibited during initial test load increments, and assuming ΔQ remains constant throughout the test, $\Delta Q/\Delta\varepsilon$ may increase over several test load increments as strain becomes compatible over the pile's total

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cross-section. Additionally, a reduction in $\Delta Q/\Delta\varepsilon$ may indicate essentially full mobilization of geotechnical resistance between a SGL and test load location due to increased $\Delta\varepsilon$, where the full change in test load is transferred to the specific strain-gauge level.

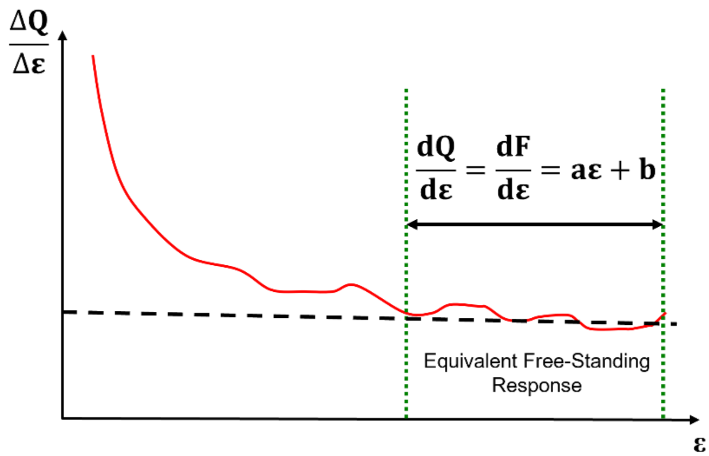

Fig. 1. Linearly resolved ΔQ/Δε plot for one interpretable strain-gauge level.

Quantitatively, a strain-dependent rigidity function can be defined as the "effective rigidity," whereas rigidity at a specific strain magnitude may not be easily defined based on conventional mechanics of fully composite materials. The term effective rigidity may be used to define the proportionality of a parameter to internal force (linear and non-linear proportionality), or conditions where strain compatibility is not a valid assumption over a load-test duration (and thus, the computed composite-section rigidity is not directly proportional to internal force).

The best-fit line coefficients from a linearly resolved $\Delta Q/\Delta\varepsilon$ plot are used to calculate internal forces at an interpretable strain-gage level according to the following relationship (Komurka & Moghaddam 2020):

$$F_i = 0.5a_i\varepsilon_i^2 + b_i\varepsilon_i + c_i \quad [2]$$

Where for each interpretable SGL_i, a_i = y-intercept of the linearly resolved $\Delta Q/\Delta\varepsilon$ plot, b_i = slope of the linearly resolved $\Delta Q/\Delta\varepsilon$ plot, and c_i is the integration constant (since at zero internal force there is zero strain, c_i is equal to zero). Therefore, the linearly decreasing effective rigidity versus measured strain resolves to:

$$E_iA_i = 0.5a_i\varepsilon_i + b_i \quad [3]$$

A review of Eq. 2 indicates that to convert measured strains to calculated internal forces at interpretable SGLs, neither a pile's composite-section elastic modulus, nor its total cross-sectional area, need be known. In this way, the I.R. method offers more direct conversion of measured strains to calculated internal forces than the approach presented in Eq. 1.

CASE HISTORY

Test Pile Details

The test pile, designated TP-1, consisted of a concreted drilled shaft, having an embedded length of 60 feet. As-built diameters varied, from 33 inches from ground surface to 12 feet (in a temporarily cased section), tapering over the next six feet to 30 inches below. The results of two unconfined compressive strength tests performed the day of the test on four-inch-diameter cylinders averaged 6,605 pounds per

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square inch ("psi"). The pile was bi-directionally statically load tested using a jack assembly with a fracture plane at a depth of 45 feet. A total of six strain-gauge levels were installed, four above and two below the jack assembly, with two strain gauges at each level mounted diametrically opposed, Fig. 2. The gauges consisted of Geokon Model 4911 36-inch-long vibrating-wire "sister-bar" rebar strainmeters constructed with #4 reinforcing bar. The strainmeters were installed between the flanges of a W16x31 core beam which was welded to the top and bottom of the jack assembly, and plunged into the fluid concrete, Fig. 3. Of interest for this case history is the measured strain response at SGLs A1 (above the jack assembly), and SGLs B1 and B2 (below the jack assembly). By inspecting these $\Delta Q/\Delta\varepsilon$ plots, other SGLs were considered uninterpretable, and the purpose of this paper is to address interpretation of atypical strain readings where feasible.

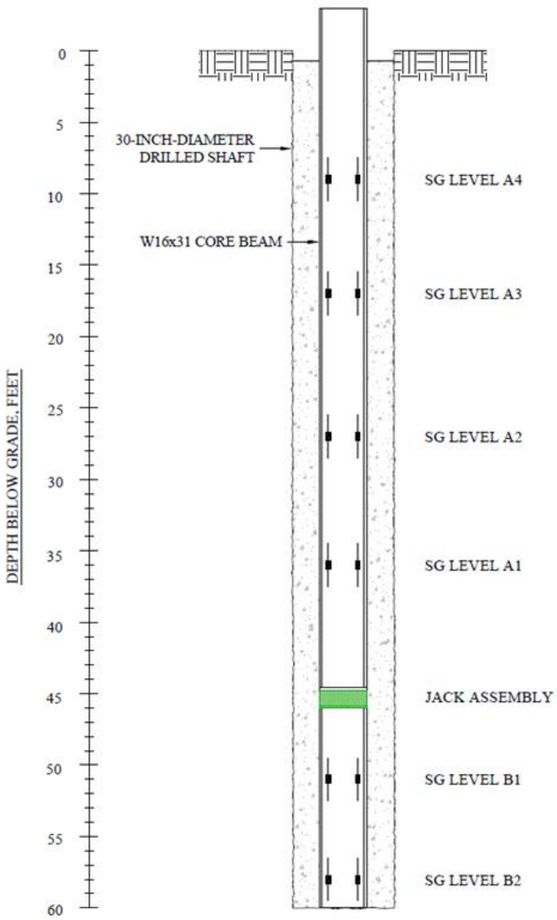

Fig. 2. Test pile schematic.

Fig. 3. Typical strainmeter installation.

Loading Sequence and Strain Responses

TP-1 was loaded in 22 load increments, and unloaded in 5 load decrements, Fig. 4. Early in the test, noteworthy upward shaft head displacement became apparent, risking significant reduction in the final number of load increments attainable. Accordingly, load increment magnitudes were decreased to increase the final number of load increments (a field decision to potentially provide better data interpretation). The strain responses from all strain-gauge levels are presented in Fig. 5.

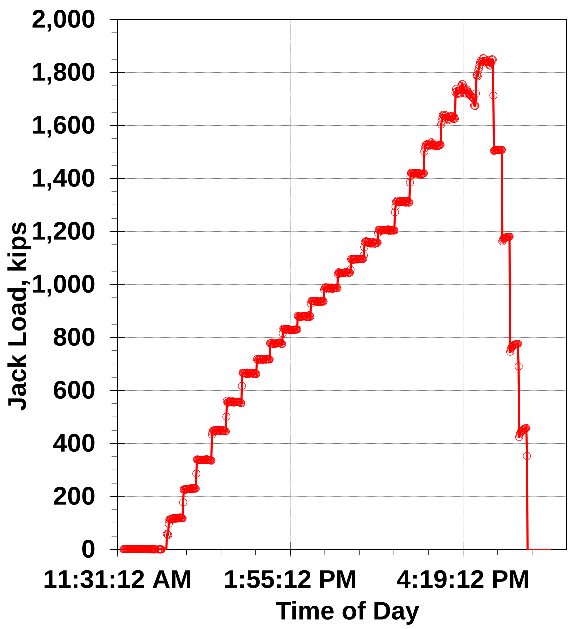

Fig. 4. Load vs. time.

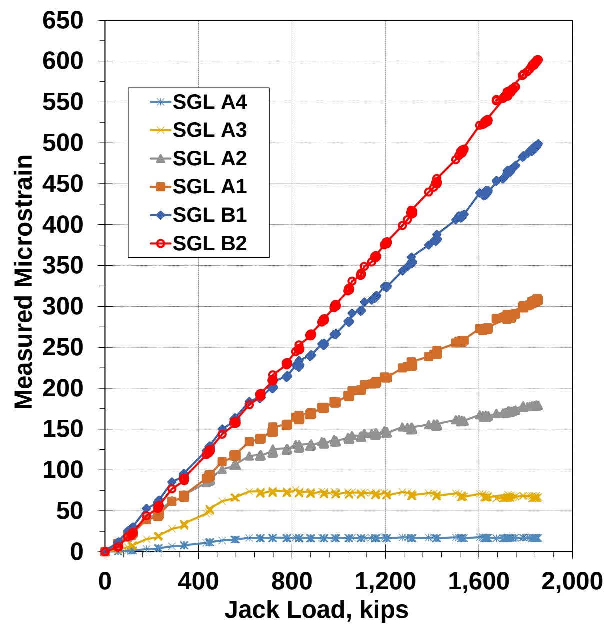

Fig. 5. Strain vs. load.

Incremental Rigidity Interpretations and Calculated Internal Forces

Incremental Rigidity evaluations presented herein are from strain readings from SGLs A1 (first level above the jack assembly), B1 (first level below the jack assembly), and B2 (second level below the jack assembly, near the shaft base).

<u>Strain-Gauge Level B2</u> – The $\Delta Q/\Delta\varepsilon$ plot using average strains for SGL B2 is presented in Fig. 6. A review of Fig. 6 indicates that the $\Delta Q/\Delta\varepsilon$ plot for B2 resolved into two distinct interpretable linear responses (i.e., bilinear response), potentially indicating a change in effective rigidity during the test. This conclusion is reinforced with the indicated increase in $\Delta\varepsilon$ over the test duration (evidenced in Figs. 5 and 6 at approximately 250 microstrain), resulting in a decrease in $\Delta Q/\Delta\varepsilon$. The slopes and intercepts of the interpreted bilinear linear responses are reported on Fig. 6 (with bilinear responses, both equations are used to calculate internal forces, each being applied to the strain ranges over which they are presented on $\Delta Q/\Delta\varepsilon$ plots). When the bilinear $\Delta Q/\Delta\varepsilon$ strain responses determined from Fig. 6 were used to calculate internal forces at SGL B2, those calculated internal forces were nearly identical to downward test loads throughout the test, Fig. 7.

<u>Strain-Gauge Level B1</u> – For B2's internal forces interpretation to be plausible, calculated internal forces at SGL B1 must also be nearly identical to downward test loads. The $\Delta Q/\Delta\varepsilon$ plot using average strains for SGL B1 is presented in Fig. 8. A review of Fig. 8 indicates that a) again the average strain readings exhibit bilinear response, and b) the resulting $\Delta Q/\Delta\varepsilon$ plots are somewhat "noisy." As at SGL B2, when the bilinear $\Delta Q/\Delta\varepsilon$ plot strain responses determined from Fig. 8 were used to calculate internal forces at SGL B1, calculated internal forces were nearly identical to downward test loads throughout the test, Fig. 9. This tends to validate the internal force interpretations at both SGLs B1 and B2. This indicates that a) the I.R. method can effectively model bilinear $\Delta Q/\Delta\varepsilon$ responses, indicative of change in effective rigidity during the test, and b) a $\Delta Q/\Delta\varepsilon$ plot can be interpreted to provide reasonable calculated internal forces even with somewhat "noisy" response, and so may not be as subjective as might be assumed.

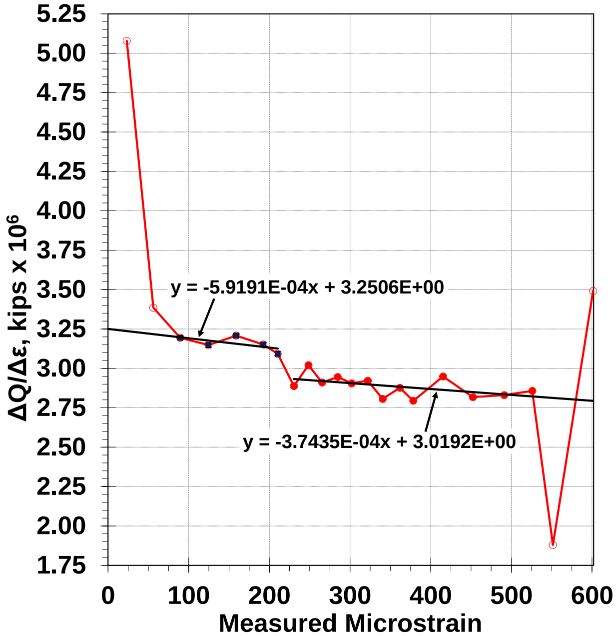

Fig. 6. Strain-Gauge Level B2 ΔQ/Δε plot.

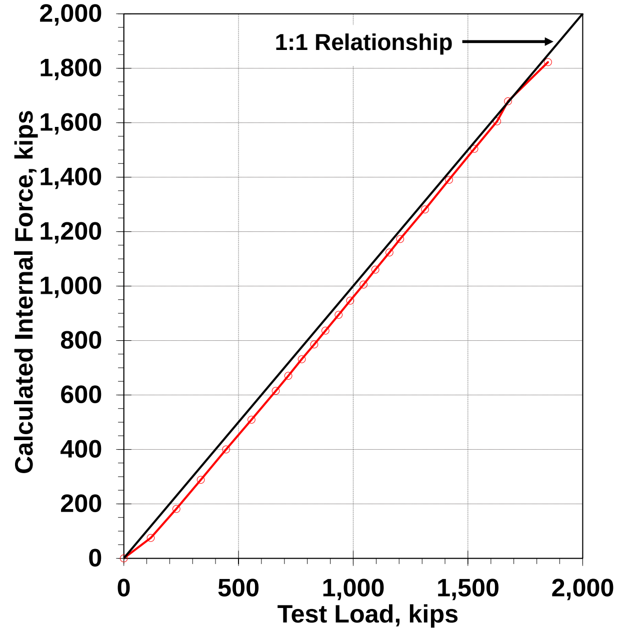

Fig. 7. Strain-Gauge Level B2 Calculated internal forces vs. downward test loads using the I.R. method and Eq. 2.

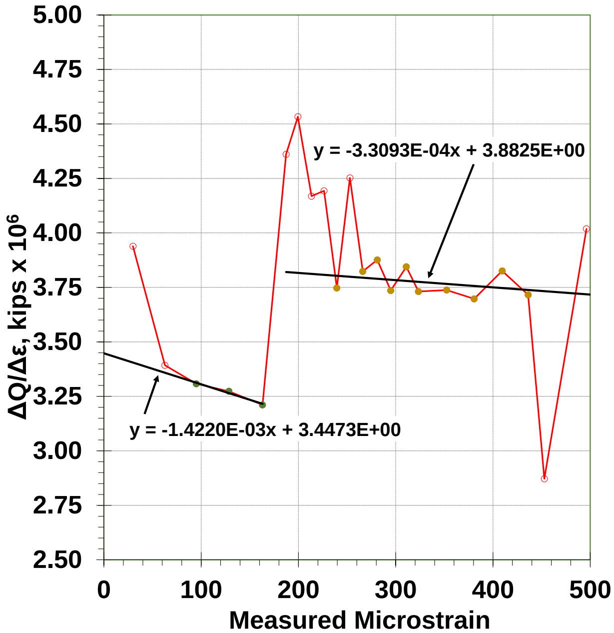

Fig. 8. Strain-Gauge Level B1 ΔQ/Δε plot.

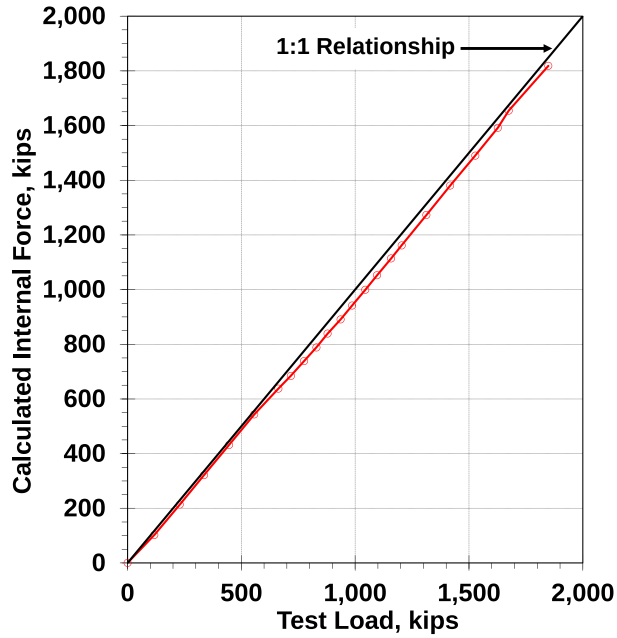

Fig. 9. Strain-Gauge Level B1 Calculated internal forces vs. downward test loads using the I.R. method and Eq. 2.

Strain-Gauge Level A1 – The ΔQ/Δε plot using average strains for SGL A1 is presented in Fig. 10. A review of Fig. 10 indicates that a) again the gauges' average strain readings exhibit bilinear response, and c) again the resulting ΔQ/Δε plots are quite "noisy." Linear interpretations of various slopes and intercepts for the average strain readings are reported on Fig. 10. As at SGLs B1 and B2, when the bilinear ΔQ/Δε strain responses determined from Fig. 10 were used to calculate internal forces at SGL A1, calculated internal forces were nearly identical to upward test loads throughout the test, Fig. 11.

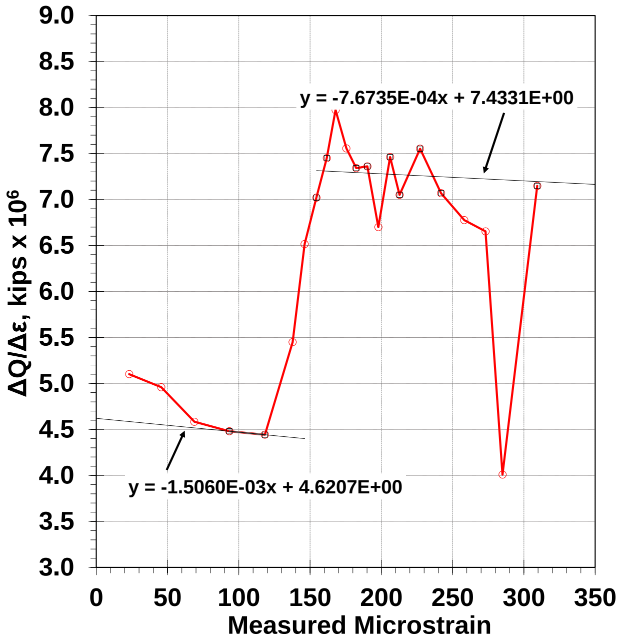

Fig. 10. Strain-Gauge Level A1 ΔQ/Δε plot.

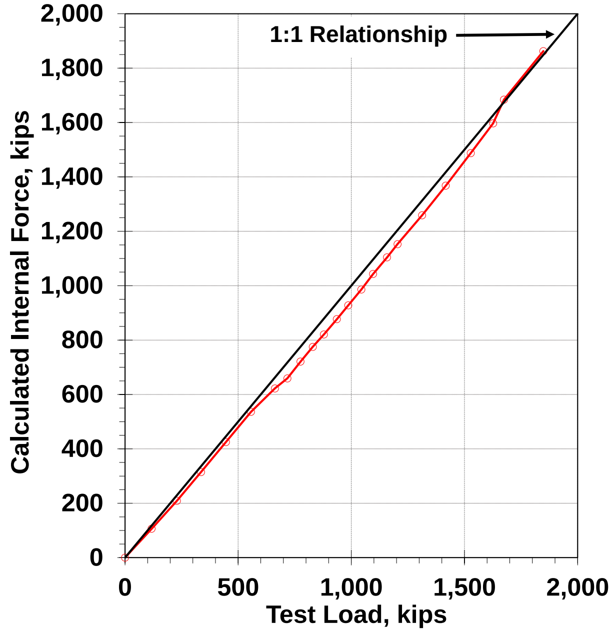

Fig. 11. Strain-Gauge Level A1 Calculated internal forces vs. upward test loads using the I.R. method and Eq. 2.

It should be noted that at all three evaluated strain-gauge levels, although strain readings yielded "noisy," bilinear ΔQ/Δε responses (i.e., experienced changes in rigidity during the test), in all cases the I.R. method accounted for these phenomena, and provided reasonable internal force interpretations.

Inherent in interpreting strain readings utilizing the I.R. method is the assumption that upstream side-shear resistance is essentially fully mobilized prior to developing the effective rigidity function. Therefore, the ΔQ/Δε interpretation could be misleading early in the loading phase, resulting in a higher axial rigidity than what would be otherwise computed if the pile were truly a free-standing column. However, this case study clearly demonstrates atypical strain readings, qualitatively identified via examining ΔQ/Δε over the test duration. Additionally, the results presented herein exhibit abnormal pile response to applied load, indicating either negligible pile-soil shear resistance in the lower foundation portion, and/or lack of composite action due to the instrumented core beam application. Further evaluation of the American Concrete Institute ("ACI") formula is presented in the following text for assessing the validity of a common interpretive method of converting measured strains to calculated internal forces.

The ACI Formula and Calculated Internal Forces

A number of empirical relationships exist to estimate concrete elastic modulus based on unconfined compression strength determined from test cylinders as reported in the American Concrete Institute ("ACI") Committee Report 363-10 (2010). One of the more popular relationships for normal-weight concrete is offered in the ACI 318-14 manual (2014), and is given by the following relationship:

$$E_{CONC} = 57{,}000(f'_c)^{0.5} \qquad [4]$$

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Where E_{CONC} = concrete elastic modulus, psi, and f'_c = concrete test cylinder unconfined compressive strength, psi. Komurka and Moghaddam (2020) provided a number of uncertainties regarding assessing the elastic modulus of concrete using the ACI formula. These uncertainties include (among others) the many ways in which the f'_c determined from concrete cylinders in the laboratory may not reflect the mass-concrete strength in the pile, (potentially owing largely to differences in curing conditions), and that Eq. 4 does not account for modulus strain dependency.

For comparison purposes, the ACI formula (Eq. 4) was used to estimate concrete modulus using the concrete test cylinders' average unconfined compressive strength. This concrete modulus, along with the pile's steel modulus and constituent concrete and steel cross-sectional areas, was used to calculate the pile's composite-section modulus. Plots of the resulting calculated internal forces vs. test loads using Eq. 1 and the constant (linear, strain-independent) concrete modulus estimated by the ACI formula (to determine the composite-section modulus) for the three SGLs are presented in Fig. 12. A review of Fig. 12, especially by comparison to Figs. 7, 9, and 11, indicates that calculated internal forces estimated using the ACI formula might have been misinterpreted as reasonable at SGLs A1 and B1. However, calculated internal forces at SGL B2 are greater than both the test loads and the calculated internal forces "upstream" (closer to the load source) at SGL B1, both of which are physical impossibilities.

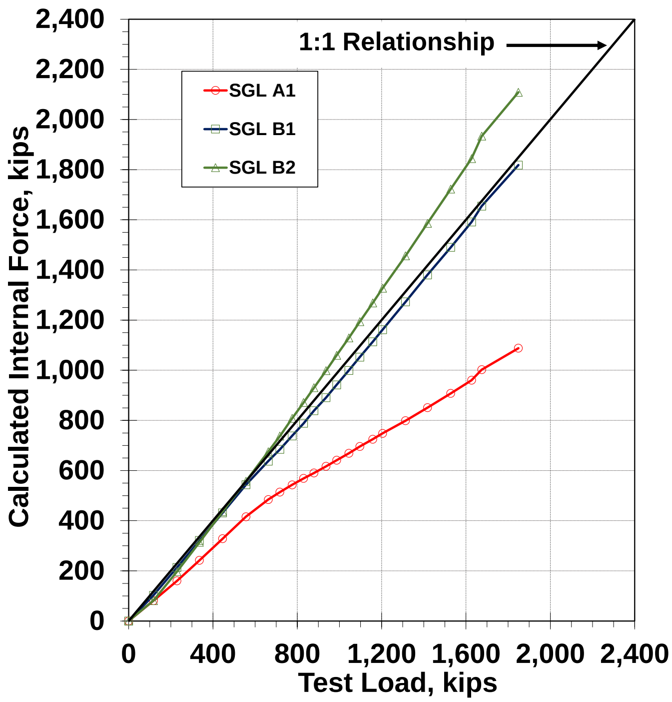

Fig. 12. Strain-Gauge Levels A1, B1, and B2 calculated internal forces vs. test load using the ACI formula (Eq. 4) and Eq. 1.

Potential Contributing Mechanisms to Bilinear ΔQ/Δε Responses

The methodology in applying the ACI formula to estimate the concrete modulus (to be used to calculate the composite-section rigidity) relies on the assumption of composite-section action and strain compatibility across the full pile cross-section at each SGL. Based on the results presented in Fig. 12, higher internal forces than the applied load are computed, as well as higher internal forces compared to an upstream location, both physical impossibilities. This suggests that either a) the ACI-formula-computed concrete elastic modulus is much greater than the actual elastic modulus, or b) composite-section action cannot be justified, which may result in a significantly lower effective rigidity. Utilizing an instrumented core beam results in a high concentration of steel directly attached (welded) to the bi-directional jack assembly. It may be inferred that stress concentrates in the steel element (i.e., the foundation element does not have compatible strain over its full cross-section). Instrumented core beams must also have adequate concrete-steel development length adjacent to the jack assembly (above and below), such that the concrete-steel interfacial resistance is sufficient to transfer applied load to the surrounding substrate materials. These inferences cannot be readily assessed from strain readings alone, and if the underlying assumptions of the ACI formula are invalid, the ACI-formula-calculated internal forces are fundamentally invalidated.

Uncertainty lies in all methods for interpreting strain readings. The internal force computations presented herein (i.e., the I.R.-method and ACI-formula solutions) may indicate that applied loads are not adequately distributed to the surrounding substrate materials. Bilinear I.R.-method solutions may be applied to strain readings for more direct (i.e., without knowing EA), and more applicable (i.e., strain-dependent response) conversion of measured strains to calculated internal forces, thus demonstrating the inefficacy of the ACI formula under these scenarios. This is particularly important where a sudden change in $\Delta\varepsilon$ is exhibited during a load test, potentially indicating a change in effective structural rigidity rather than a change in resisting geotechnical forces. While the bilinear $\Delta Q/\Delta\varepsilon$ plot may not fully encompass the structural response to applied load, the I.R. method provides a reasonable assessment of the effective rigidity function over the load-test duration.

CONCLUSIONS

A general overview of the Incremental Rigidity method was presented for converting measured strains to calculated internal forces during a static load test with atypical strain readings. A case history presented interpretation of strain readings from a bi-directional static load test on a 30-inch-nominal-diamter concreted drilled shaft. The strain gauges were mounted in diametrically opposed pairs at various levels on a W16x31 core beam which was welded to the top and bottom of the jack assembly. Measured strains were converted to calculated internal forces at two SGLs below, and one SGL above, the jack assembly using the Incremental Rigidity method. At all three SGLs, the $\Delta Q/\Delta\varepsilon$ plots exhibited bilinear response, indicating a change in rigidity during the test. At two SGLs, the $\Delta Q/\Delta\varepsilon$ plots exhibited somewhat "noisy" responses. The I.R. method accounted for these phenomena, and provided reasonable internal force interpretations in all cases; application of the ACI formula did not. From this case history, the following conclusions are drawn:

- Mounting sister-bar strainmeters on a core beam for a bi-directional static load test risks obtaining atypical strain readings, and so should be avoided.
- At interpretable strain-gauge levels, the I.R. method can convert measured strains to calculated internal forces without knowledge of either a pile's composite-section modulus, or its total cross-sectional area. Limited uncertainty remains in defining an exact solution due to the mechanisms of load transfer during a static load test.
- The I.R. method can identify, and account for, a pile's change in rigidity during a static load test (i.e., model bilinear $\Delta Q/\Delta\varepsilon$ response) to yield reasonable calculated internal forces.
- The values presented on a $\Delta Q/\Delta\varepsilon$ plot ($\Delta Q/\Delta\varepsilon$ vs. ε) contain inherent inaccuracies resulting from uncertainties in test loads, strains, and potentially the stress history of both the pile itself and surrounding material. The I.R. method accounts for these inaccuracies by developing an "effective rigidity" (potentially not the pile's full composite rigidity), an internal force-strain relationship that accounts for these inherent inaccuracies, and provides reasonable internal force interpretation.
- Using the ACI formula to determine a constant (linear, strain-independent) concrete modulus used to convert measured strains to calculated internal forces provided unreasonable calculated internal forces; using the I.R. method is superior to using the ACI formula in this respect, particularly for atypical strain responses.

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