

# **DETERMINATION OF INTERNAL FORCE DISTRIBUTION USING THE INCREMENTAL RIGIDITY METHOD ON INSTRUMENTED STATIC LOAD TESTS IN INDIA**

Siddharth K. Ambaliya, Assistant Engineer, Geo Dynamics, Vadodara, Gujrat, India

E-mail: [siddharth.ambaliya@geodynamics.net](mailto:siddharth.ambaliya@geodynamics.net)

Sujan Kulkarni, General Manager, Geo Dynamics, Vadodara, Gujrat, India

E-mail: [sujan@geodynamics.net](mailto:sujan@geodynamics.net)

Seth Robertson, Ph.D., Project Engineer, GRL Engineers, Inc., Cleveland, USA

E-mail: [srobertson@grlengineers.com](mailto:srobertson@grlengineers.com)

## **ABSTRACT**

Conventional static load tests can be complemented with instrumentation, such as strain gauges and tell-tale displacements, to calculate the geotechnical resistance distribution along an embedded pile's length in skin friction and end bearing. Measured strains are typically multiplied with theoretical area and Elastic Modulus (i.e., EA, the axial rigidity) to calculate internal forces and ultimately the geotechnical resistance distribution. However, for bored piles/drilled shafts, pile area may not be uniform and elastic modulus of concrete is strain dependent. Hence, use of empirical formulae or use of above-grade strain gauges for elastic modulus estimation may lead to erroneous load transfer estimation. A more-direct method to estimate force transfer distribution is use of the Tangent Modulus method, as proposed by Fellenius (2001), which eliminates some of the limitations in estimation of Elastic Modulus, yet still requires a reasonable estimation of the cross-sectional area at each strain gauge location. This limitation can be improved if Axial Rigidity (EA) is estimated and used directly in calculation of force transfer distribution, defined as the Incremental Rigidity method. This paper reviews the various methods available for estimating the internal force distribution from instrumented static load tests, and presents case studies comparing estimated load transfer distribution with different methods.

**Keywords: Instrumented Pile Load Tests, Concrete Elastic Modulus, Load Transfer, Axial Rigidity, Incremental Rigidity**

## **INTRODUCTION**

Reinforced concrete bored piles or drilled shafts are commonly used foundation systems across the globe. With increase in structural loads, demand of heavily-loaded pile foundations is on the rise. The typical pile foundation installation consists of mainly three operations which are drilling the specified diameter borehole up to design depth, lowering the reinforcement cage, and concreting the borehole up to the designated cutoff level. Depending on subsurface conditions and size of pile foundations, drilling operations are performed with augers, hydraulically operated rotary rigs, or with Top-drive Reverse Circulation Drill (RCD) technique [1]. Sometimes, temporary or permanent casings are also used to avoid collapse of upper soft soil into the hole. Various types of fluids are also used to stabilize the sidewalls by maintaining positive hydrostatic head to minimize potential for soil collapse. Typically, water or water mixed with bentonite or polymer, is used as stabilizing drilling fluids. After successful completion of drilling operations, the borehole is flushed for removal of soft sediments accumulated at the bottom, and then the reinforcement cage is lowered into the open excavation. For wet drilling operations, concrete is placed via a tremie pipe discharged at the bottom of the drilled hole. The concrete for piling shall be designed such that it provides adequate workability, stability, durability, and strength. Once the pile casting is completed, several tests can be performed to assess pile integrity and its load bearing capacity. The commonly used non-destructive integrity testing methods are Pile Integrity Testing ("PIT"), Cross-hole Sonic Logging testing ("CSL"), and Thermal Integrity Profiling ("TIP").

Static load testing (“SLT”) is the most trusted method to assess the load bearing capacity of the piles in order to confirm the pile design. High Strain Dynamic Pile Testing (“HSDPT”) and Bi-directional Static Load Testing (“BDSLTL”) are also popular and commonly adopted load test methods. However, static load test offer the major advantage over HSDPT and the BDSLT as it simulates the loading pattern as expected under service loading conditions. For most conventional SLT, load is applied gradually in equal increments, and the pile top settlement/displacement is measured to establish the pile’s load-settlement characteristics. In most Indian scenarios, the load is applied either incrementally in a single cycle (commonly known as maintained loading) or in multiple cycles (known as cyclic loading). Quick load tests in which loading increments are applied in quick succession and are performed in other parts of world, however these approaches are not common in India. The reaction for static load tests can be obtained from either kentledge, reaction piles, ground anchors, or a combination of these. For heavier loads it is common to obtain reaction from reaction piles or ground anchors.

For a conventional top-down static load test, load is applied to the pile head and is resisted from the head to base by means of skin friction and end bearing resistances. Since a conventional SLT measures only applied load and pile top settlement, there is no information on load transfer characteristics or quantification of mobilized skin friction and end bearing resistances. IS 2911-Part 4 (2013) [2] outlines a method to estimate skin friction and end bearing resistance from cyclic load test data, however it is an indirect method resulting in limited reliability of the overall computations. The better, and more direct, way to estimate load transfer characteristics is by installing strain gauges at various locations along the depth of the pile. Strain gauge levels can be designated at locations of subsurface stratigraphy changes, near known cross-sectional changes, or equi-spaced along the pile’s embedded length. Proximity to applied load should also be considered due to potential incompatible strains over the composite cross-section. Therefore, strain gauges are generally recommended to be at least one diameter (preferably two diameters) from the applied load source. Near or above-grade instrumentation can provide means of back-calculating composite material properties (i.e., Secant-Modulus method), however the internal calibration sequence may not necessarily apply to below grade instrumentation.

Measured strains can further be converted to force transferred along the embedded pile length [3]. Unit skin friction can then be determined by dividing the computed boundary forces by the unit areas of each foundation segment. It should be noted, however, due to variability in existing methods for computing internal force from strain measurements, and the inherent subjectivity and uncertainty in existing methodologies, there is no direct way to compute “exact” resistance distribution along an embedded foundation length. Practitioners must therefore rely on proper engineering justification to determine the most reasonable skin friction and end bearing characteristics from load test results. Subsequent sections highlight the procedure for computing internal force distribution for instrumented static load tests and describe the complexities associated with various interpretative methods.

## **STRAIN TO INTERNAL FORCE**

Static load tests can be complemented with embedded instrumentation such as strain gauges for estimation of internal force distribution. Strain gauges are installed at various levels along the depth of the pile, and when load is applied at the pile head, distribution of internal strain is measured. For a uniform pile, strain gauges undergo strains proportional to the force transferred at the strain gauge location. Hence upper strain gauges (closer to applied load) will generally undergo maximum strains, and bottommost strain gauges will undergo minimum strains. Once the strain measurements at a particular level are determined, then the internal force at that particular level can be calculated as,

$$P = EA\varepsilon \quad [1]$$

Where E = Composite Elastic Modulus, A = Total area of the pile, and  $\varepsilon$  = Measured strain.

Here, strain is measured directly, but the concern arises that pile area is generally considered uniform with depth, and elastic modulus of concrete is considered constant over the strain history during the SLT. Existing methods for estimating concrete Elastic Modulus varies within a wide range, and is also understood to be strain-dependent [3, 5]. Hence if indirect methods are used to estimate Elastic Modulus, then the conversion of strain to internal force becomes complicated and the interpretation is dependent upon accurate pile installation records. As specified in IS 456:2000 [4], E is computed as,

$$E \text{ (MPa)} = 5000 * (f_{ck})^{0.5} \quad [2]$$

Where  $f_{ck}$  = Characteristic Cube compressive strength (Mpa).

The code also specifies that the actual measured values of E may differ by  $\pm 20\%$  from the values computed using the above equation. Since Equation 2 is not a direct measurement of concrete elastic properties, it is defined as an empirical methodology for estimating Elastic Modulus. An improved method is therefore warranted to address the aforementioned inherent issues.

Internal calibration of strain gauges to applied load can be better estimated using Incremental Rigidity method [5]. For strain to load conversion, as represented in Eq. 1, elastic modulus (E) and area (A) shall be known. However, it is not necessary to know the individual quantity. As far as product EA, if “Axial Rigidity” is known, the equation can be resolved to compute internal force. Hence,

$$P = \varepsilon R \quad [3]$$

Here, R is rigidity and is a product of the composite E and A.

The concept was originally proposed by Fellenius as Tangent Stiffness or Tangent Modulus method [6]. Fellenius proposed to plot the tangent modulus versus strain for each loading increment and at each level of strain gauge measurement. The tangent modulus is calculated as changes in stress divided by changes in strain. A best fit line is drawn from the tangent modulus versus strain plot, and the equation of this best fit line is used to estimate the secant modulus at each representative strain reading. This strain-dependent secant modulus is further used in equation 1 to estimate the force transferred at various strain gauge levels. The tangent modulus method addresses many shortcomings of the empirically calculated elastic modulus as it takes into account the strain dependency of the elastic modulus.

Komurka and Moghaddam [5], understood the fact that independent quantities E and A are not required for internal force computation, and hence re-defined the “Tangent Stiffness” method as the Incremental Rigidity (I.R.) method. Alternative to assessing changes in stress versus changes in strain versus strain, the I.R. method suggests plotting change in applied load to strain to provide more-direct conversion of measured strain to internal force. The plot results are a straight line sloping from higher to lower rigidity (i.e., strain-dependent rigidity). Mathematically, I.R. can be represented as

$$\frac{dF}{d\varepsilon} = a\varepsilon + b \quad [4]$$

$$F = \int (a\varepsilon + b) d\varepsilon \quad [5]$$

$$F = 0.5A \varepsilon^2 + B \varepsilon + C \text{ (where } C \text{ is assumed } 0) \quad [6]$$

$$F = (0.5A \varepsilon + B) \varepsilon \quad [7]$$

From Eq. 7 it is evident that force transferred can be calculated without knowing  $E$  and  $A$  independently. The strain gauge level nearest the applied load location plays an important role, as it is most likely the strain measurement that is least affected by the external resisting forces.

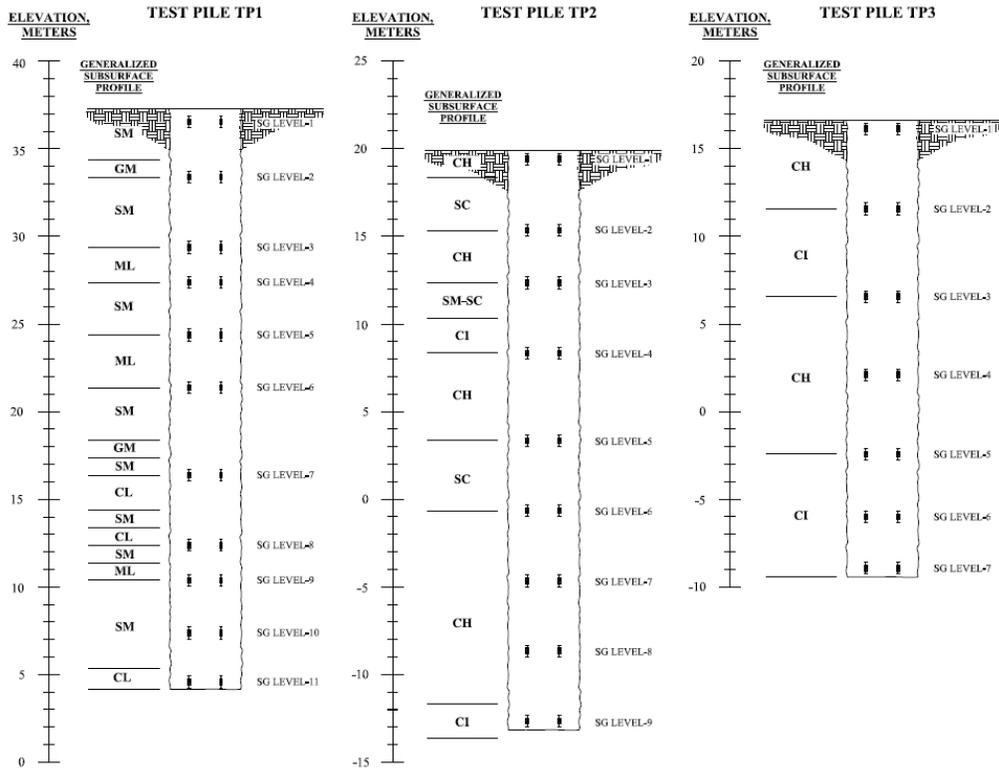
Since the internal force is determined at each embedded strain gauge level, and is generally known at the foundation head as the applied test load, the foundation is divided into segments bounded by the ground surface, each embedded strain gauge level, and the pile base. Unit shaft resistances are then computed as the difference in boundary forces divided by the surficial area for each foundation segment.

Komurka and Robertson discuss critical aspects for proper interpretation of strain measurements for applying the I.R. method [8]. For example, the method is only theoretically applicable where geotechnical resistance between applied load and a particular strain gauge level is fully mobilized. Therefore, practitioners must rely on pile installation records, subsurface conditions, and field observations during testing (e.g., pile displacements and creep behavior) to reasonably justify that nominal soil resistance has been fully activated prior to applying the I.R. method. To promote I.R. application and provide interpretable results, strain gauges should not be located too close to a load source or directly at known foundation rigidity changes. The loading apparatus (e.g., hydraulic jack) should be of sufficient capacity to fully activate geotechnical resistances over the majority foundation length. The method is best suited when loads are uniformly applied, using equally stepped load, and each load is maintained over equal durations to minimize non-uniformities in strain response over the duration of the loading procedure.

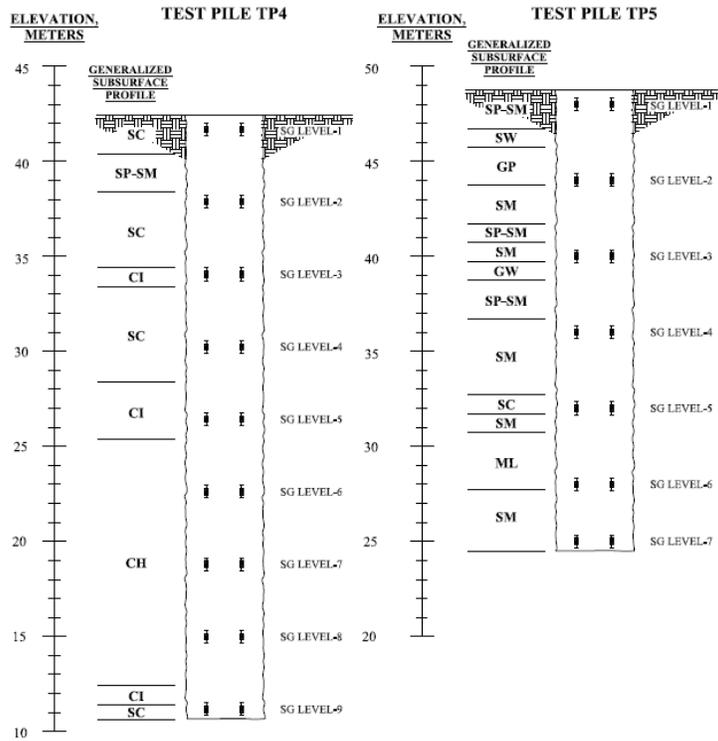
## **INSTRUMENTED STATIC LOAD TEST CASE HISTORIES**

Several case histories are presented to demonstrate the importance of proper conversion of measured strain to internal force. The static load tests followed the maintained load and cyclic loading procedures. However, unload/reload cycles and non-uniform test-load holds can make interpretation of I.R. relationships more difficult, and is generally ill-advised particularly if strain gauge data is critical in the foundation design [5]. As maintained and cyclic loading designations remain common practice (particularly in India and other surrounding regions), a closer examination of these static load test methods is required. Several comparisons of empirical correlations to determine Elastic Moduli and analyses using the I.R. method are therefore presented.

The presented case histories were from two projects conducted around Gujarat, India. Table 1 presents the general project details. Test piles were 1500mm and 1800mm diameter bored piles, and ranged from 24 to 33 meters embedment. Substrate materials were generally clayey and silty/clayey sands. Project 1 included three test piles, and the static load tests were conducted using the maintained loading method with non-uniform load-hold durations. Project 2 included two test piles, and the static load tests were conducted using the cyclic loading method. Fig. 1 and Fig. 2 present elevation views of each test pile with the strain gauge layouts for Project 1 and Project 2, respectively



**Fig. 1. Pile elevation views and instrumentation layout for Project 1**



**Fig. 2. Pile elevation views and instrumentation layout for Project 2**

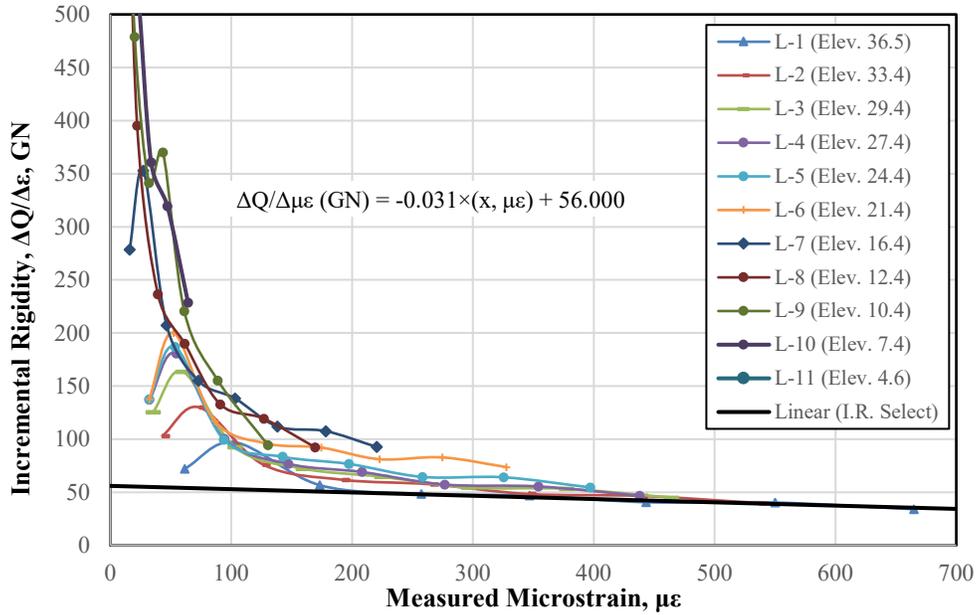
An example of the determined I.R. relationship is presented in Fig. 3 from Project 1; TP1. Fig. 4 presents a comparison of the calculated internal force distribution for Project 1; TP1. Strain gauges are numerically designated based on their level, and numerically increases with further distance from the applied load location (i.e., pile head).

A comparison of the calculated rigidities using the aforementioned empirical formulae (and nominal areas) and I.R. method are presented in Table 1. The average uniaxial compressive strength is reported for cubic concrete samples taken during test shaft installation. It should be noted that no concrete samples were tested for Project 2, and the reported concrete compressive strength is a generalized reported strength based on the concrete mix used for the particular project. This observation extends concerns that empirical correlations between concrete compressive strength and Elastic Modulus may not be warranted due to inadequate information provided for load test data reduction. Additional uncertainty arises from the SLT methodology, magnitude of applied load, and strain gauge locations specified on a project, which may inherently nullify the internal calibration sequence of strain gauges for use of the I.R. method.

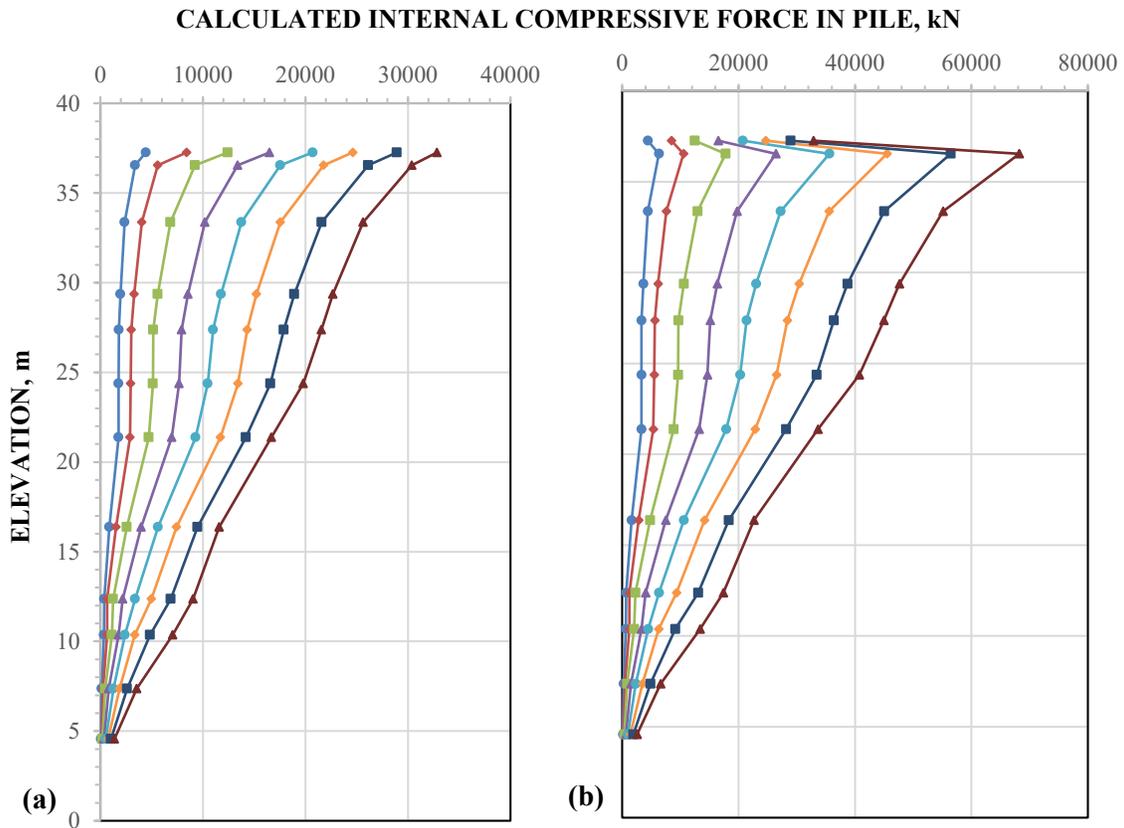
**Table 1. Static load test case history summary**

Test Pile Designation	Project 1; TP1	Project 1; TP2	Project 1; TP3	Project 2; TP4	Project 2; TP5
Concrete Grade	M40	M40	M40	M40	M40
Uniaxial Compressive Strength (MPa)	65	59	57	50*	50*
Pile Diameter, mm	1800	1800	1500	1500	1500
Pile Depth, m	33.1	33.1	26.1	31.5	24.0
Maximum Applied Test Load, kN	32800	33500	23500	22000	22000
Maximum Strain ( $\mu\epsilon$ ) (SG Level-1 @ Max Loading)	665	852	581	482	435
Calculated Rigidity (Empirical), GN	102.6	97.7	66.7	62.5	62.5
Calculated Initial Rigidity (I.R. Method), GN	56.0	46.0	45.0	46.0	56.0
Calculated Internal Force @ SG Level-1 and Max Loading, kN – (Empirical)	68210	86810	38740	30115	27210
Calculated Internal Force @ SG Level-1 and Max Loading, kN – (I.R. Method)	30395	32355	22675	21300	21735

\*Specific compressive strength tests were not performed on concrete samples for these test piles. An approximate strength reported for other foundations installed on this project is therefore used.



**Fig. 3. Incremental Rigidity plot from Project 1; Test Pile TP1**



**Fig. 4. Comparison of the calculated internal force in the pile versus elevation from Project 1; Test Pile TP1 using (a) the I.R. method, and (b) empirical method**

## DISCUSSION AND FINDINGS FROM CASE HISTORIES

The calculated internal forces displayed in Fig. 4 demonstrate the inherent issues with establishing a constant axial rigidity, particularly based on empirical correlations between concrete compressive strength and concrete Elastic Modulus in determining a single representative axial rigidity. Internal forces near the ground surface in Fig. 4 exceed the applied loads, which is an impossibility in mechanics of materials. Therefore, direct comparison of calculated skin friction and end bearing resistances cannot be established between the two methods for these case histories. Summarized in Table 1, the computed internal forces in strain-gauge level 1 (nearest the ground surface) at the maximum load present similar findings for each case. On an additional note, the computed rigidities based on empirical methods were determined from the calculated concrete Elastic Modulus multiplied by nominal area, and are thus of lower magnitude than compared to the steel-reinforced composite properties.

Several uncertainties in empirical methods provide credence to these findings:

1. Concrete Elastic Modulus is known to be strain-dependent. Establishing a uniform value will lead to misinterpretation of computed internal forces over the full range of measured strain.
2. There is a large spread in existing empirical correlations between concrete compressive strength and Elastic Modulus. The existing formulae may provide reasonable results for some concrete mix designs, but may not be reasonable for others. Elastic Modulus is sensitive to aggregate inclusion, depending on aggregate type, percent fraction, and size. Therefore, establishing a single formula applied to all concrete mixes and grade is not reasonable.
3. Cubic concrete samples have been found to provide higher compressive strength compared to an equivalent cylindrical concrete sample [7]. This observation may account for an increased computed Elastic Modulus on cubic samples when using correlations established from cylindrical samples.
4. Concrete samples taken above grade, cured in a semi-controlled environment, and tested unconfined in a laboratory setting are not indicative of the in-place concrete qualities.
5. Bored piles are prone to geometric variation versus depth. These non-uniformities will directly affect the pile's axial rigidity at a strain gauge location. Methods for direct geometric measurement over an embedded pile length are extremely limited (arguably non-existent).
6. Where concrete samples are not obtained for a particular test pile, concrete Elastic Modulus cannot be empirically or directly determined. Engineering justification for determining internal force from measured strain is thus practically limited.

Empirical correlations to determine Elastic Modulus are oftentimes unreasonable. Therefore, embedded strain measurements may not be meaningful. Improved computed internal force distributions, and in turn geotechnical resistance distributions, is realized with the I.R. method. The I.R. relationship displayed in Fig. 3 was determined from the uppermost strain-gauge level. This relationship was applied to all other strain-gauge levels. A similar procedure was developed for each case history. Representative strains were established at the end of each virgin loading increment, which is more critical for cyclic loading procedures. The results presented herein demonstrate that, despite the complexities, I.R. can be utilized for maintained and cyclic loading conditions with certain considerations. The case histories were conducted by the same test practitioners, and special care was taken to maintain applied load and to keep proper records over the duration of each load test. If applied pressure is not maintained over a load duration, and record of test measurements is not reported, significant uncertainty in the representative test measurements for each loading increment is realized. Proper selection of representative applied loads and measured strains for each loading increment is a critical element of interpreting the computed I.R. relationships.

Assessment of the I.R. relationship at each individual strain gauge level may be warranted for some cases [8]. A single representative relationship was established from each case history for comparison purposes. Since the I.R. method is only valid where soil resistance is fully mobilized between the applied load and

particular strain gauge level of interest, non-interpretable measurements may be realized from the I.R. plot in lower strain-gauge levels (with increasing uncertainty further from load application). This attribute is exhibited in Fig. 1, where higher intercepts and steeper slopes are seemingly apparent if a fit line is applied to all individual strain gauge levels. This is likely due to lack of full resistance mobilization at each particular strain gauge level (e.g., levels L-4 to L-11 in Fig. 3), resulting in reduced  $\Delta\mu\epsilon$  compared to an equivalent free-standing column (and thus increased  $\Delta Q/\Delta\mu\epsilon$ ). A near-grade strain gauge level can improve I.R. interpretation by minimizing external soil resistance influences. However, special considerations should be taken due to potential loss of strain compatibility near the load application. It is generally recommended to maintain strain gauges at least one-diameter distance, and preferably two diameters, away from the applied load location. Back-calculated elastic moduli can additionally be applied to non-interpretable strain gauge levels if additional pile properties are reasonably estimated, such as implementing TIP or other non-destructive profiling methods for estimating pile area versus depth [9].

For static load test projects where interpretation of embedded strain measurements is critical, geotechnical and structural engineering designers should consider specifying Elastic Modulus laboratory tests in lieu (and/or in conjunction) with uniaxial compressive strength tests. Consistent load durations, and stepped loading increments can also significantly improve the data reduction process. Ultimately, a successful static load test and data reduction is dependent upon, but not limited to, local experience with soil stratigraphy, proper load scheduling, location of embedded instrumentation, and methods of conducting the test with respect to applied load and data acquisition.

## CONCLUSIONS

Elastic Modulus computation for subsequent internal load transfer evaluation using empirical equations shall be avoided as it can result in erroneous estimates. Elastic Modulus is not a constant value, but it is a strain-dependent quantity. The Incremental Rigidity method addresses this issue, and it is an appropriate approach for estimation of internal force distribution when complemented with proper engineering justification.

Several case studies were presented to demonstrate uncertainty in various methods for reducing embedded strain gauge measurements. Based on the information provided and the results presented, the following conclusions can be made:

1. The internal force within a foundation element, and subsequent determination of resistance distribution, under static axial load cannot be easily measured directly. Correlations between measured strain and internal force is the current industry standard. Interpretation of embedded strain measurements for computation of internal forces requires proper data review and justification of the particular analytical methods implemented.
2. Empirical correlations between concrete uniaxial compressive strength and Elastic Modulus are generally used as the industry standard for determined concrete elastic properties. Comparisons of calculated pile axial rigidity determined from the I.R. method and empirical correlations were presented from five static load tests. The empirical correlations consistently resulted in unreasonable internal force profiles (i.e., calculated internal forces that did not comply with theoretical mechanics), and therefore could not be applied for determining the resistance distribution along the embedded pile lengths. The I.R. method provided significantly improved internal force profiles, utilizing interpretable measurements from near-grade strain gauges.
3. Various methods of applying static load can complicate the analytical procedures to compute the internal force distribution. The case histories were conducted using the maintained and cyclic load test methods which are commonly employed in India. While it is generally advised to use consistent stepped loads and durations for optimizing strain gauge data reduction, the studies presented demonstrate feasible application for the I.R. method under maintained and cyclic load tests.

4. Improved industry standards can be achieved if Elastic Modulus tests are specified in conjunction with uniaxial compressive strength tests on concrete samples. Cylindrical concrete samples are preferred for compressive strength tests in some parts of the world, and may be required for Elastic Modulus test requirements and empirical correlations. Maintained and cyclic load tests may not yield interpretable strain measurements for implementing the I.R. method. Quick load tests, in which loading increments are applied in relatively shorter succession and smaller increments than maintained load tests, are preferred where computation of internal force from embedded strain is critical. Proper designation of the maximum applied load (at or near the ultimate geotechnical capacity), and locality of embedded strain gauges, will significantly improve the interpretation of strain measurements.

## REFERENCES

- [1] Brown, D.A., Turner, J.P., Castelli, R.J., and Loehr, E.L. 2018. Drilled Shafts: Construction Procedures and Design Methods, Geotechnical Engineering Circular No. 10, FHWA Report No. FHWA-NHI-18-024, National Highway Institute, U.S. Department of Transportation, Federal Highway Administration, Washington DC.
- [2] Indian Standard, IS: 2911 (Part 4) – 2013. Code of Practice for Design and Construction of Pile Foundations, Part 4 Load Test on Piles.
- [3] Fellenius, B.H. 2001. From Strain Measurements to Load in an Instrumented Pile. Geotechnical News Magazine, Vol. 19, No.1, pp 35-38.
- [4] Indian Standard, IS: 456-2000. Code of Practice for Plain and Reinforced Concrete.
- [5] Komurka, V.E., and Moghaddam, R.B. (2020). “The Incremental Rigidity Method – More-Direct Conversion of Strain to Internal Force in an Instrumented Static Loading Test.” ASCE GeoCongress 2020, Minneapolis, MN.
- [6] Fellenius, B. H. (1989). “Tangent Modulus of Piles Determined from Strain Data,” ASCE, Geotechnical Engineering Division, the 1989 Foundation Congress, F.H. Kulhawy, Editor, Vol. 1, pp. 500-510.
- [7] Elwell, D. J., Fu, G. (1995). Compression Testing of Concrete: Cylinders Vs. Cubes. Special Report 119 submitted to the Transportation Research and Development Bureau, New York State Department of Transportation, Albany, NY, March 1995.
- [8] Komurka, V.E., and Robertson, S. (2020). “Results and Lessons Learned from Converting Strain to Internal Force in Instrumented Static Loading Tests Using the Incremental Rigidity Method.” ASCE GeoCongress 2020, Minneapolis, MN.
- [9] Belardo, D., Robertson, S., Coleman, C. (2021). “Interpretation and Evaluation of Thermal Integrity Profiling Measurements”, DFI 46th Annual Conference on Deep Foundations: Las Vegas, NV, October 2021.